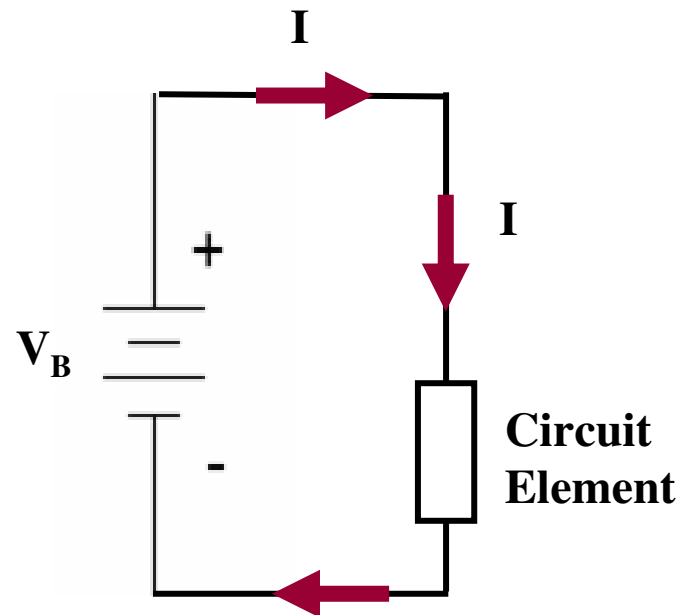


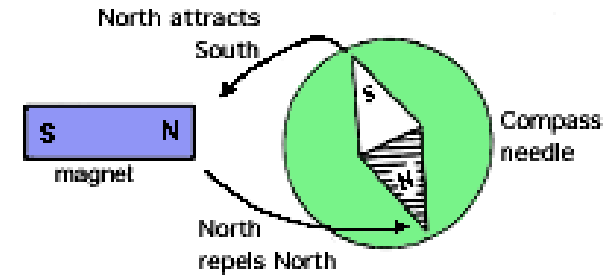
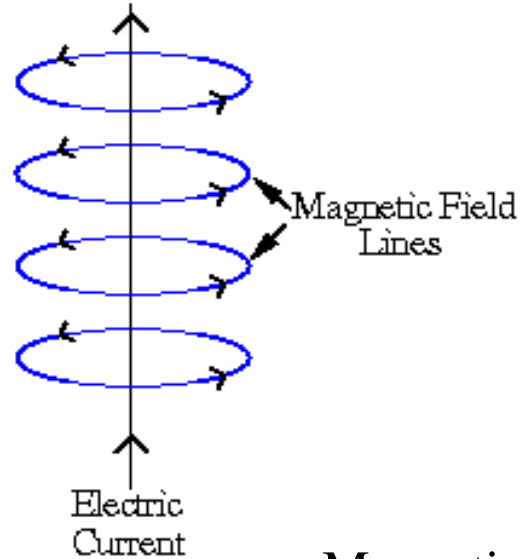
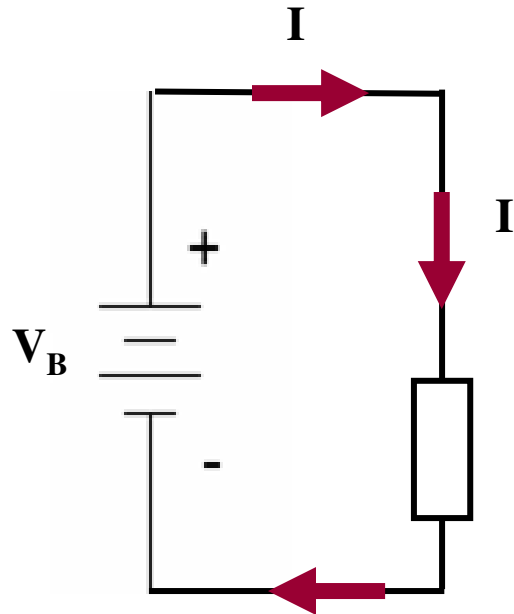
# Current and Voltage Measurements

## Current measurement

According to current continuity (i.e. charge conservation) law, the current can be measured in any portion of a single loop circuit.



## Contact-less methods



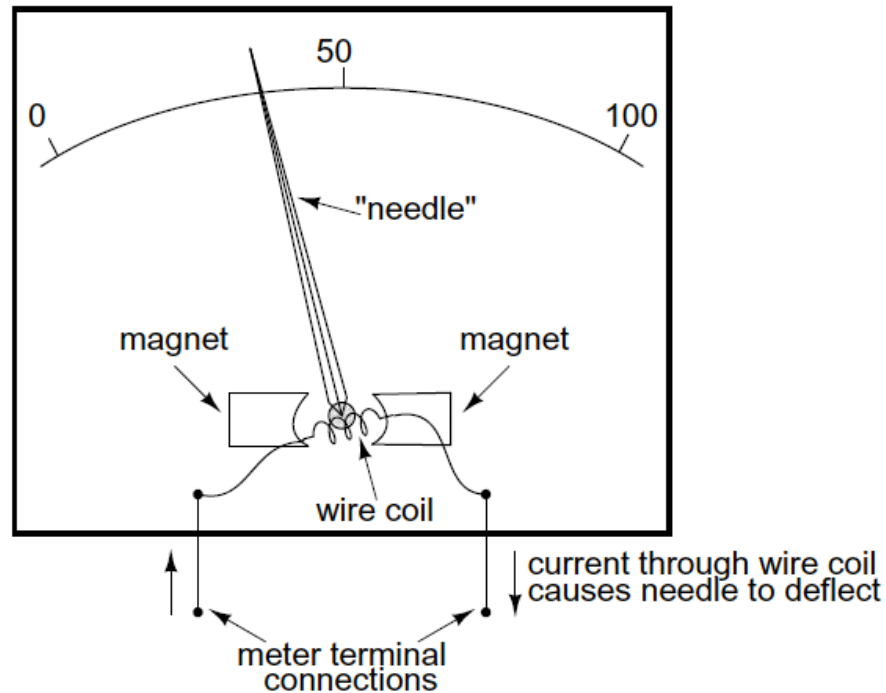
Magnetic field rotates the indicator  
(like a compass needle)

Any current produces a magnetic field.

Issues: Not very accurate. Low-sensitive – only applicable to high-current circuits

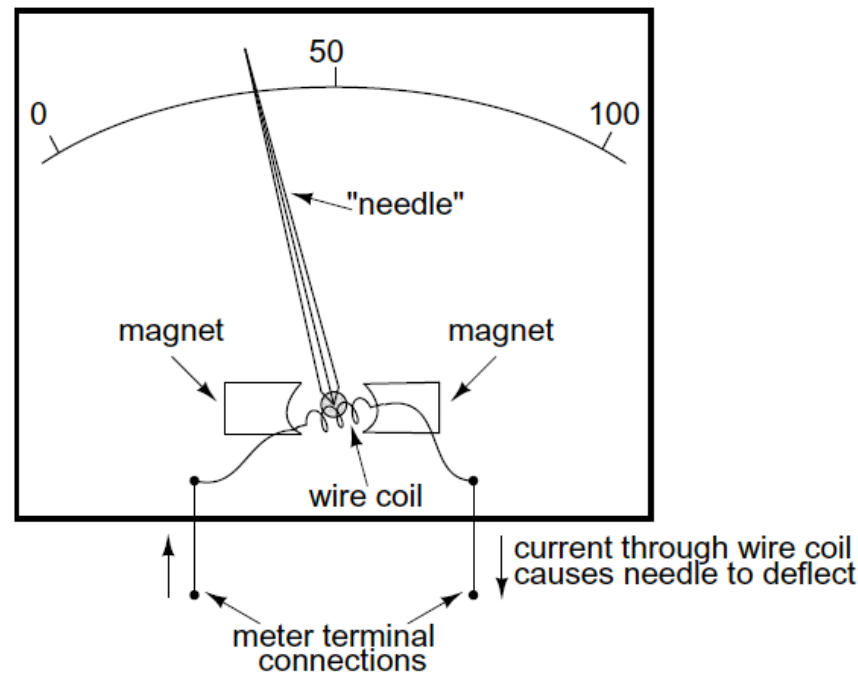
# Current measurement device – Ammeter

## Magneto-electric ammeter



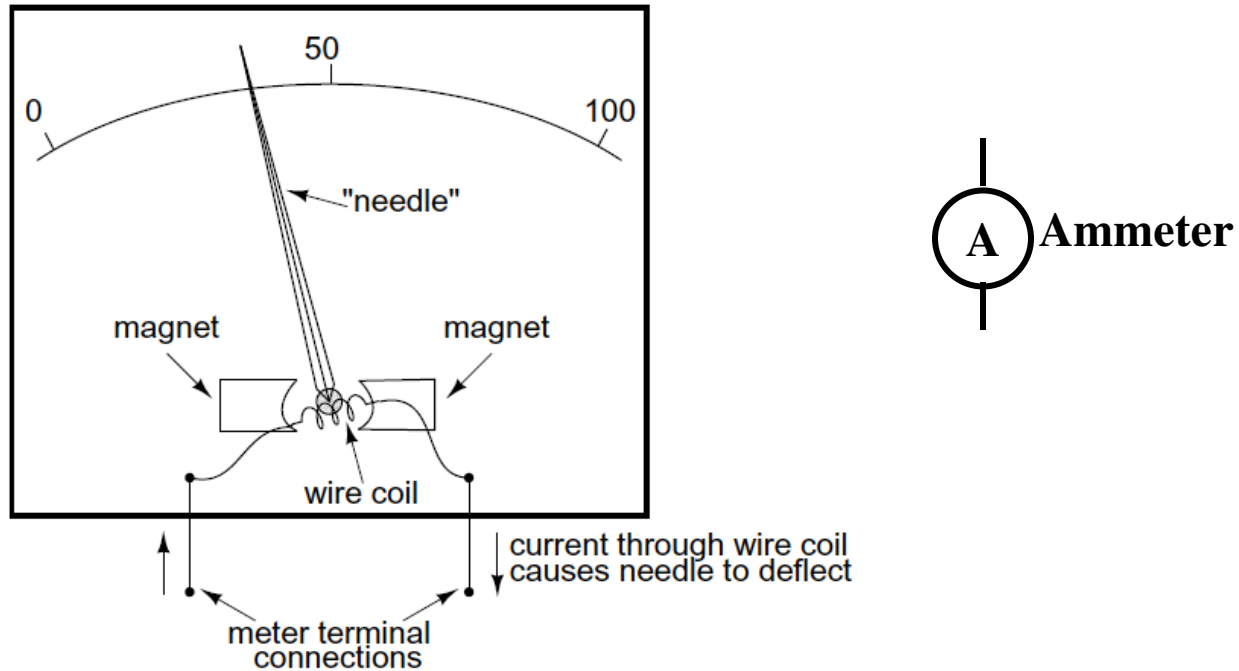
The ammeter uses the magnetic field created by the current to deflect the needle. The coil attached to the needle increases the magnetic field of the current.

## Current measurement device - Ammeter



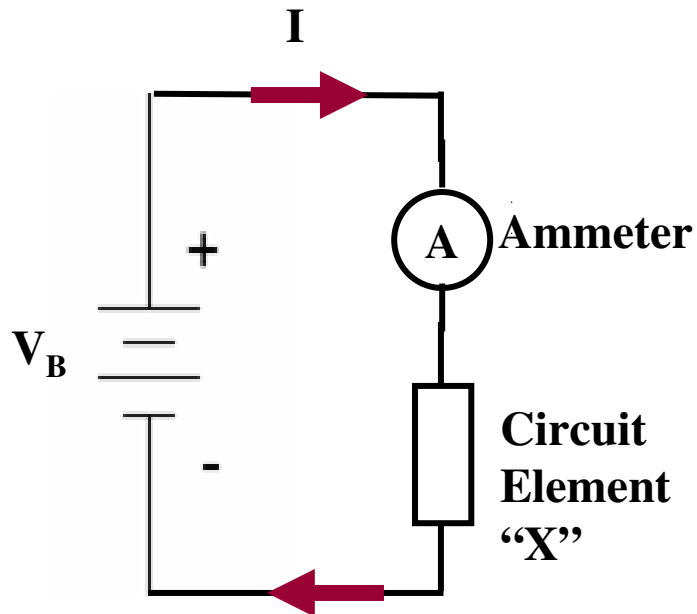
- The magnetic field of the coil is proportional to the electric current
- The deflection angle is proportional to the magnetic field
- Hence, the deflection angle is proportional to the current
- The maximum current that can be measured depends on the full scale deflection (f.s.d) of the ammeter

## Current measurement device - Ammeter



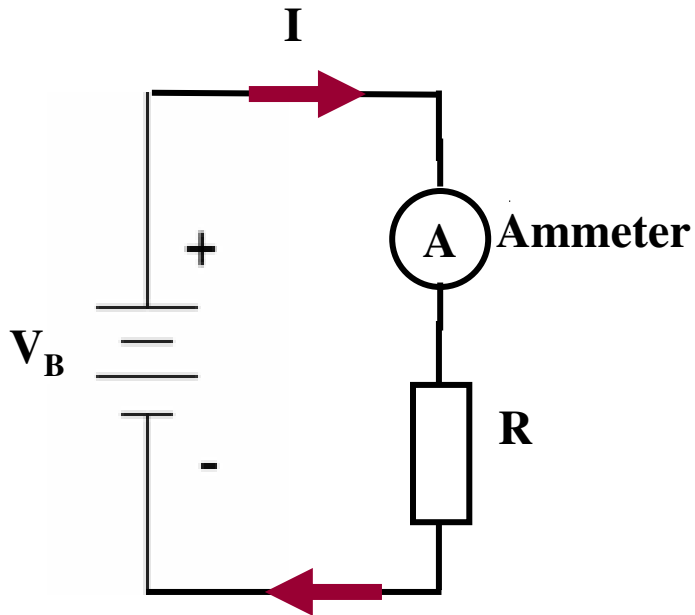
**On a circuit diagrams, an ammeter is shown as an A in a circle.**

## Series resistance method – most commonly used in all modern instruments



1. Break the circuit (or a branch)
2. Insert an ammeter
3. The current through the ammeter is measured by the needle deflection angle.
4. The current through the resistor or any circuit component X is the same as that through the ammeter
5. Electronic instruments are used to display or transmit the results

## Distortions introduced by Measurement



*The resistor  $R$  is shown by rectangle here to emphasize that the method can be applied to ANY circuit component, not to resistors only.*

The ammeter inserted in the circuit changes the actual current in it.

Let the resistance of the Element “X” be

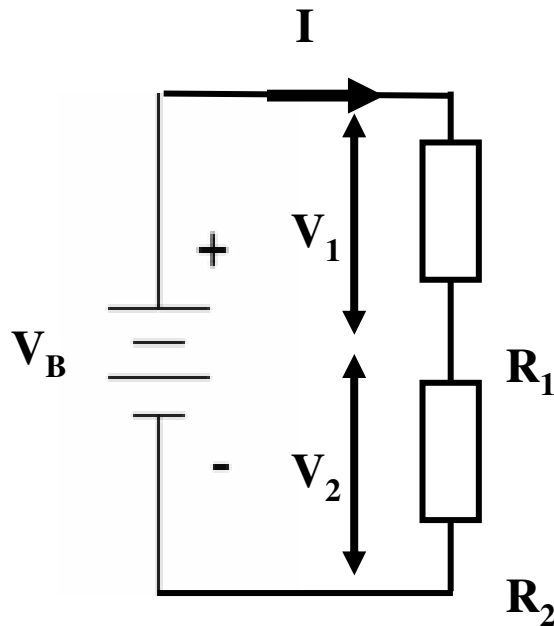
**R**

The actual circuit current is

$$\mathbf{I = V_B/R}$$

After insertion of the ammeter, the total circuit resistance changes and so does the current.

## Reminder: Series electric circuit



For the resistor 1:  $I_1 = V_1/R_1$ ;

For the resistor 2:  $I_2 = V_2/R_2$ ;

From the current continuity:

$$I_1 = I_2 = I;$$

$$V_1 = I \times R_1; V_2 = I \times R_2 \quad (1)$$

From the energy conservation:

$$V_B = V_1 + V_2; \quad (2)$$

From (1) and (2):

$$V_B = I \times R_1 + I \times R_2 = I \times (R_1 + R_2) \quad (3)$$

Solve (3) for the current

$$I = \frac{V_B}{R_1 + R_2}$$

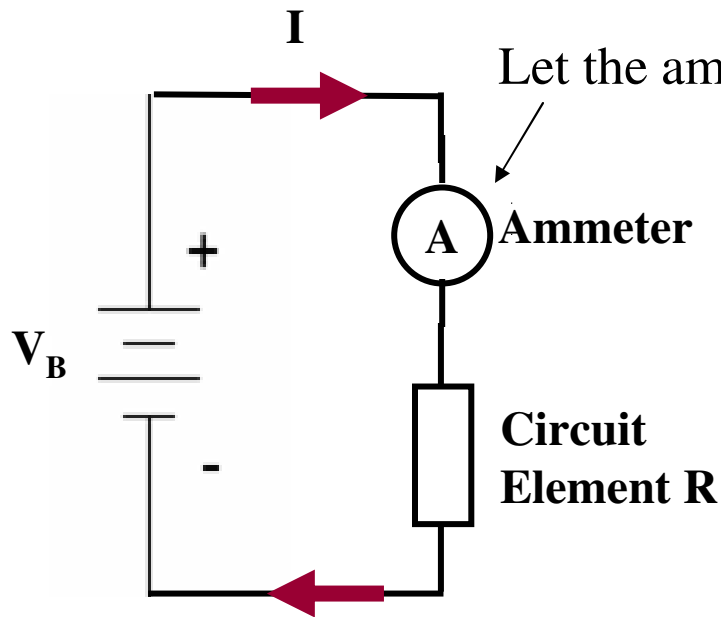
Find  $V_1$  and  $V_2$  from (1)

$$V_1 = I \times R_1 = R_1 \frac{V_B}{R_1 + R_2} = V_B \frac{R_1}{R_1 + R_2}$$

$$V_2 = V_B \frac{R_2}{R_1 + R_2}$$



## Distortions introduced by current measurement



Let the ammeter resistance (the resistance of the coil) be  $R_A$

The actual circuit current is

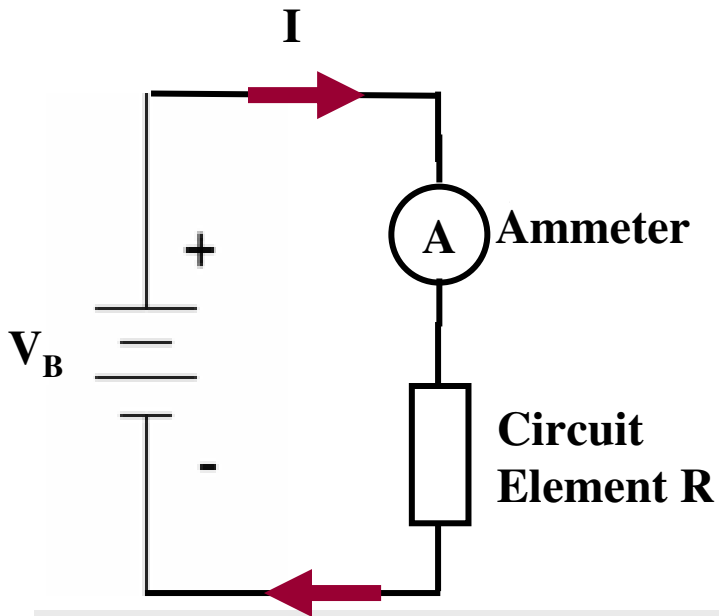
$$I = \frac{V_B}{R}$$

The measured current,

$$I_M = \frac{V_B}{R + R_A} < I$$

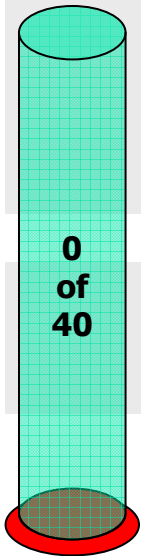
The measured current is less than the actual current in the circuit

# Problem 1



The resistor  $2\text{ k}\Omega$  is connected to a  $9\text{ V}$  battery. The Ammeter connected in series with the resistor indicates the current  $4\text{ mA}$  flowing through the circuit.

**What is the actual current in the original circuit (without the ammeter) in mA?**



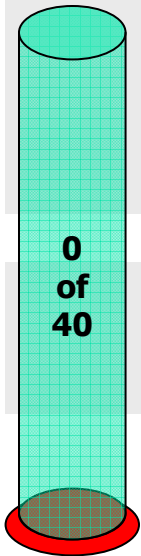
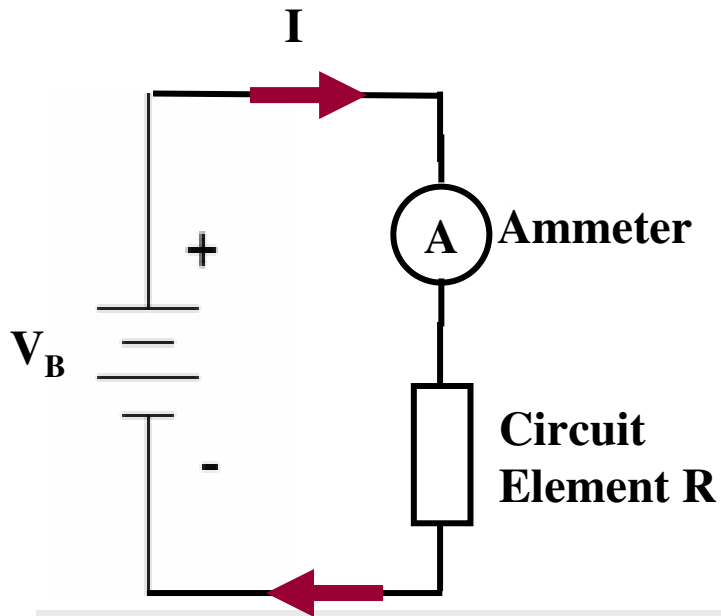
*Timed response*



## Problem 2

The resistor  $2\text{ k}\Omega$  is connected to a  $9\text{ V}$  battery. The Ammeter connected in series with the resistor indicates the current  $4\text{ mA}$  flowing through the circuit.

**What is the Ammeter internal resistance in Ohms?**

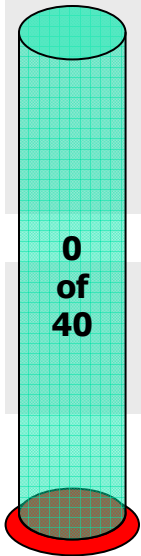
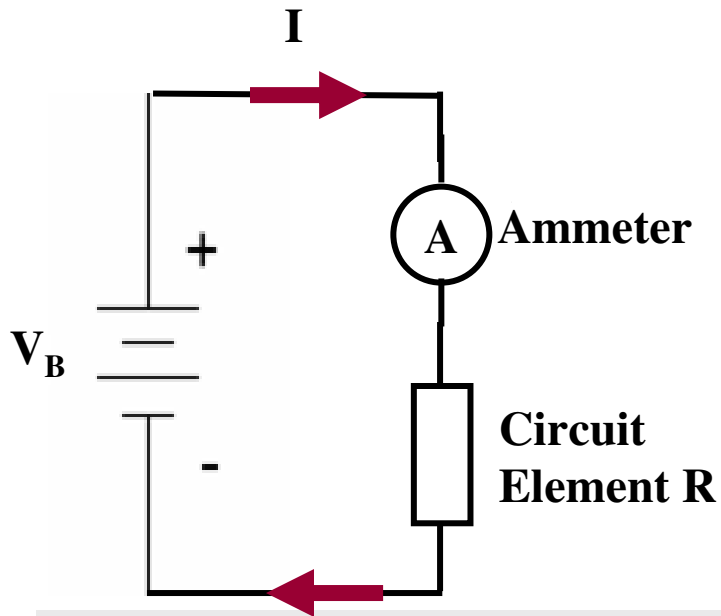


*Timed response*



## Problem 3

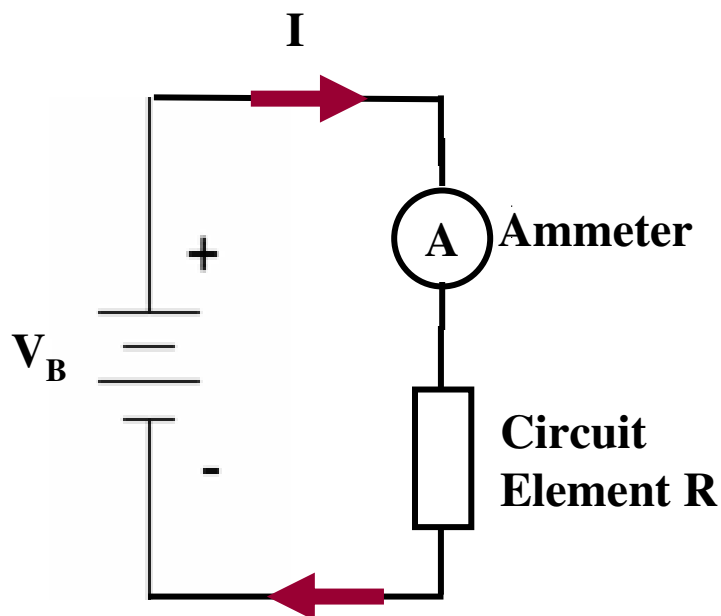
A resistor is connected to a 9V battery. The Ammeter connected in series with the resistor indicates the current 4.5 mA in the circuit. The Ammeter internal resistance is 100 Ohm. **What is the actual current in the circuit (without the ammeter) in mA?**



*Timed response*



## Distortions introduced by current measurement



The actual circuit current is

$$I = \frac{V_B}{R}$$

The measured current

$$I_M = \frac{V_B}{R + R_A} < I$$

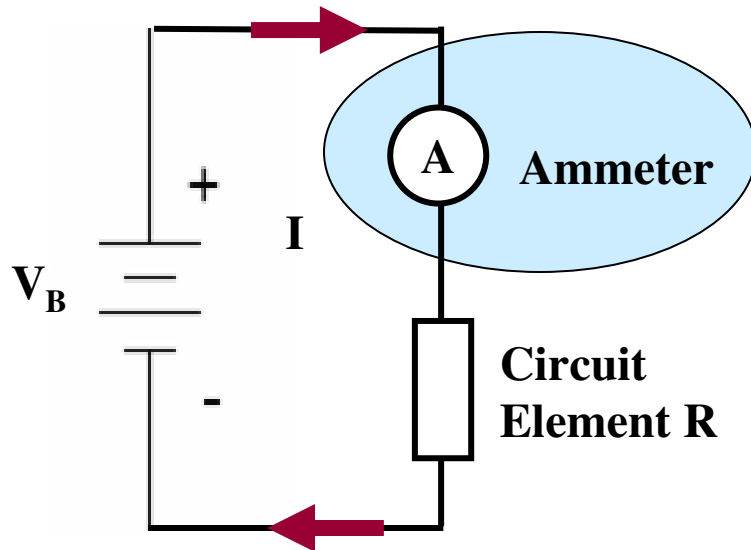
The actual / measured current ratio

$$\frac{I}{I_M} = \frac{V_B}{R} / \left( \frac{V_B}{R + R_A} \right) = \frac{R + R_A}{R} = 1 + \frac{R_A}{R}$$

The relative measurement error

$$\frac{\Delta I}{I_M} = \frac{I - \bar{I}_M}{I_M} \approx \frac{I}{I_M} - 1 = \frac{R_A}{R}$$

## Requirements for accurate current measurements



$$\frac{\Delta I}{I_M} \approx \frac{R_A}{R}$$

**For accurate measurements,  
The AMMETER resistance must be  $\ll R$**

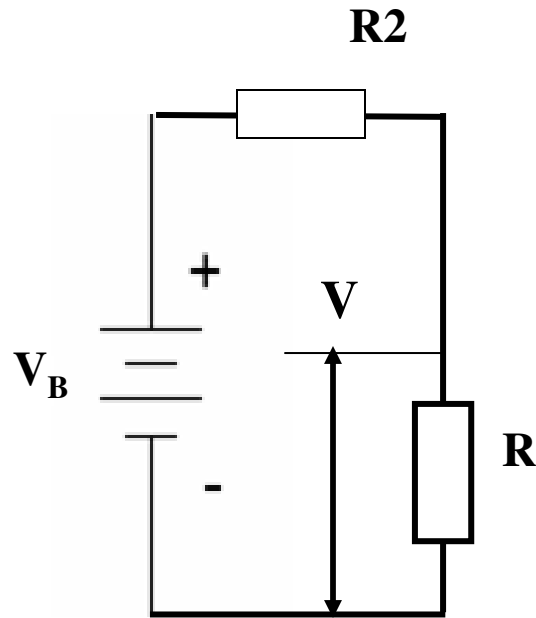
### Example

What is the relative current measurement error if the ammeter resistance

$$R_A = 0.001 \times R?$$

$$\frac{\Delta I}{I_M} \approx \frac{R_A}{R} = 0.001 = 0.1\%$$

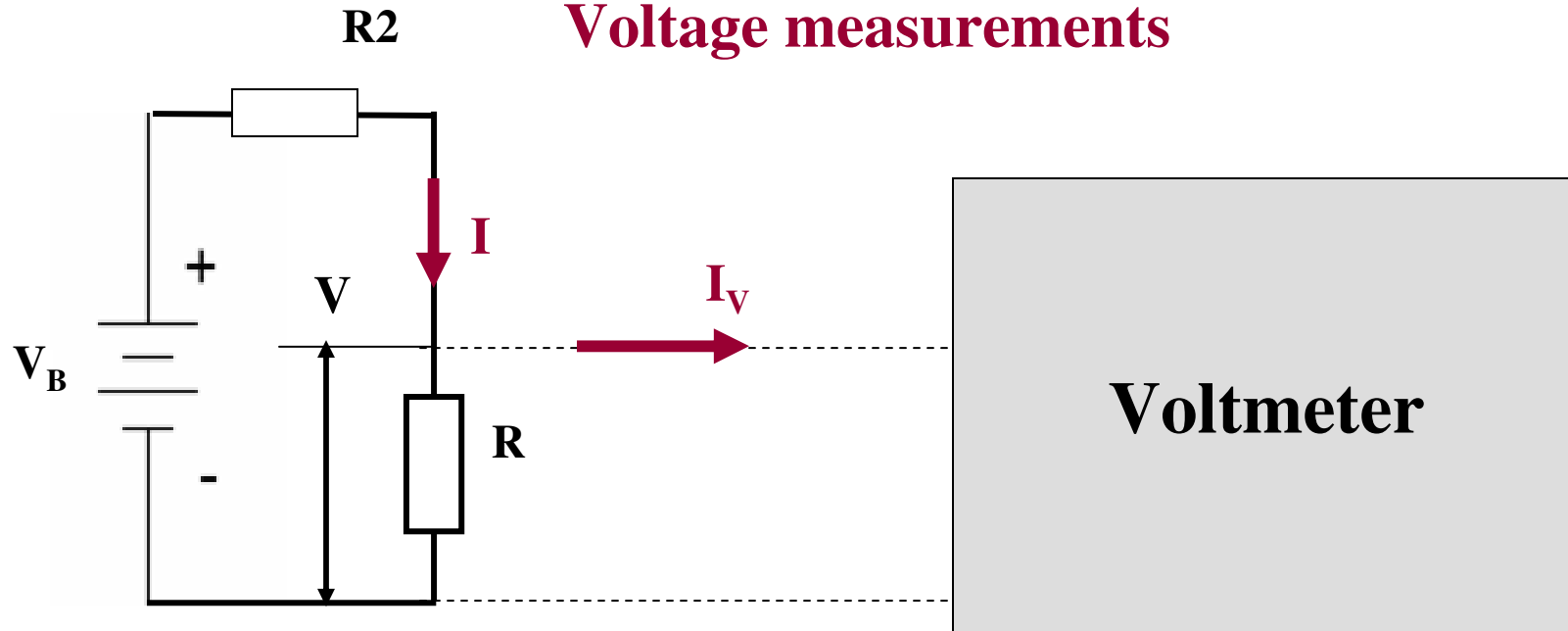
## Voltage measurements



- To measure the voltage, we need to *probe* the voltage between two nodes in the circuit.
- Various types of analog and digital electronics instruments are capable of amplifying or attenuating the input signals.
- Since voltage measurement does not need any “intrusion” it does not seem to make distortions

**NOT TRUE!**

## Voltage measurements



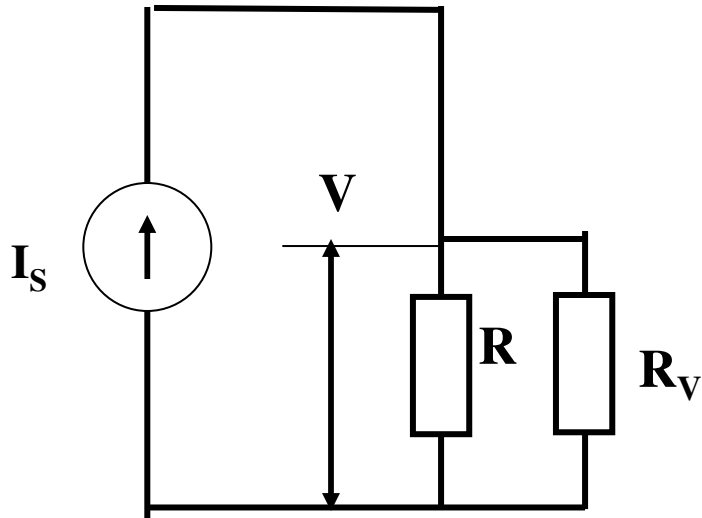
When the voltage is applied to the input of any instrument, there is a current  $I_V$  flowing into the input.

According to the charge conservation law, this current reduces the current through the test element.

Therefore, the voltage  $V$  changes, too.



## Voltage measurements



Assume the voltmeter internal resistance  $R_V$ .

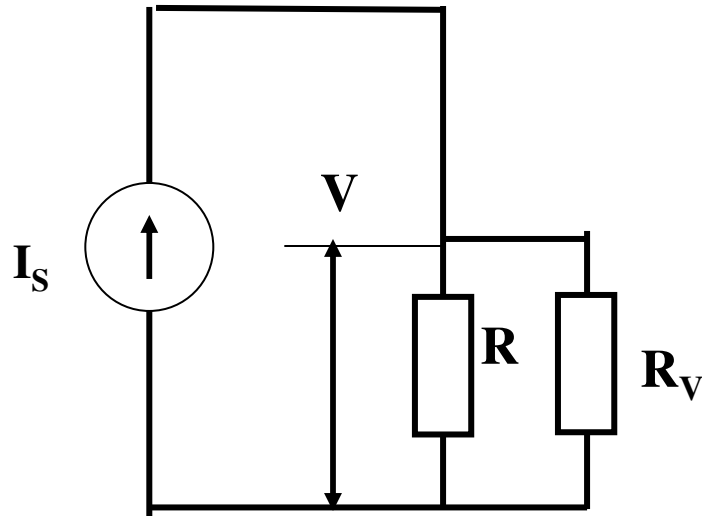
The total resistance with the voltmeter connected,

$$R_{EQ} = \frac{R \times R_V}{R + R_V} < R$$

The voltage measured by the voltmeter,  $V_M = I_S * R_{EQ}$

The actual voltage,  $V = I_S * R > V_M$

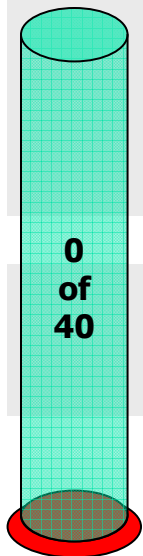
## Problem 4



Resistor 1k is connected to a 10 mA current source.

The voltmeter connected in parallel with the resistor indicates the voltage 9.5 V

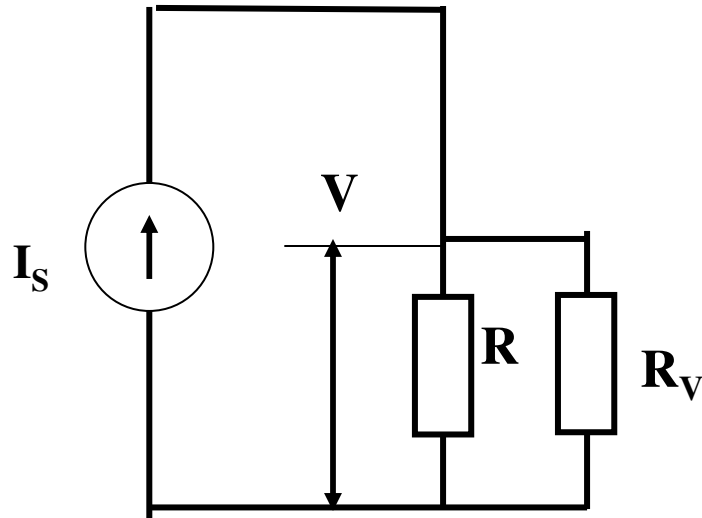
**What is the actual voltage across the resistor (with no voltmeter connected) in V?**



*Timed response*

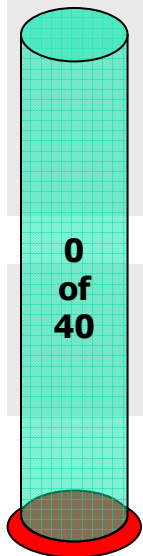


## Problem 5



Resistor 1k is connected to a current source. The voltmeter with internal resistance 10k connected in parallel with the resistor indicates the voltage 9.09 V

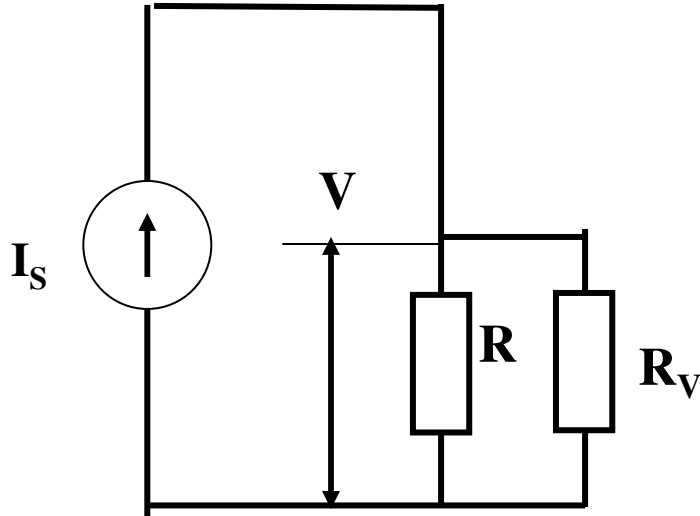
**What is the actual voltage across the resistor (with no voltmeter connected) in V?**



*Timed response*



## Voltage measurements



The actual voltage:  $V = I_s \times R$

The measured voltage:

$$V_M = I_s \times R_{EQ} = I_s \frac{R \times R_v}{R + R_v}$$

$$\frac{V}{V_M} = \frac{I_s (R + R_v) R}{I_s R \times R_v} = \frac{R + R_v}{R_v} = 1 + \frac{R}{R_v}$$

The error introduced by the measurement:

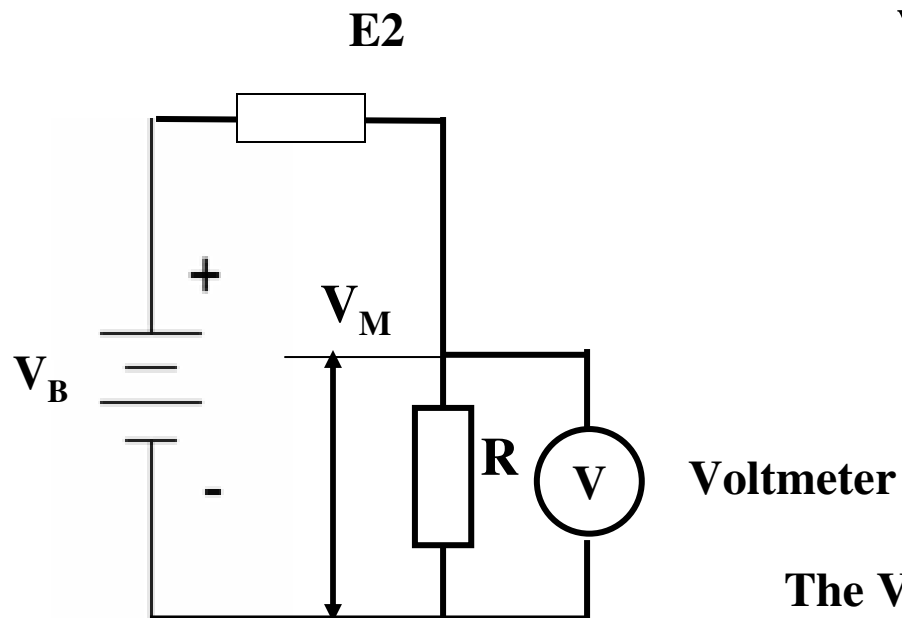
$$\frac{\Delta V}{V_M} = \frac{V - V_M}{V_M} = \frac{V}{V_M} - 1;$$

$$\frac{\Delta V}{V_M} = \frac{R}{R_v}$$

## Voltage measurements

$$\frac{\Delta V}{V_M} = \frac{V}{V_M} - 1 = \frac{R}{R_V}$$

In order to achieve less than 0.1% error in measurements, what values of  $R_V$  is needed?



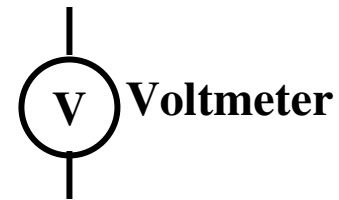
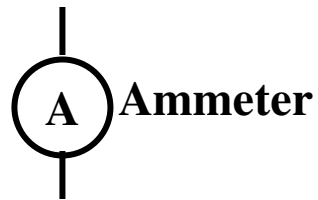
$$\frac{R}{R_V} < 0.1\% = 0.001;$$

$$R_V > 1000 \times R.$$

The VOLTmeter resistance must be  $\gg R$

On a circuit diagram, a voltmeter is shown as an V in a circle.

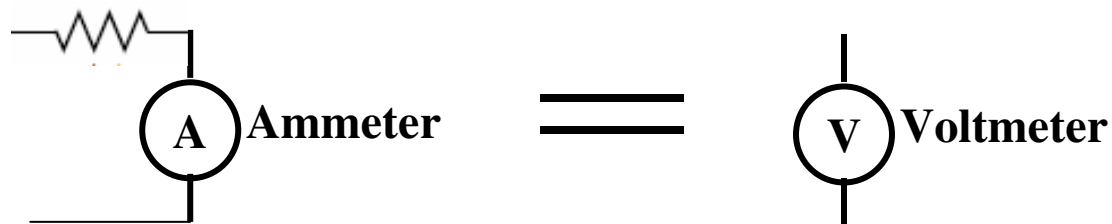
## Voltmeter and ammeter comparison



The ammeter resistance  $R_A$  must be much lower than the resistance  $R$  of the circuit to be measured

The voltmeter resistance  $R_V$  must be much higher than the resistance  $R$  of the circuit to be measured

Ammeter connected to a large series resistor can be used as a voltmeter



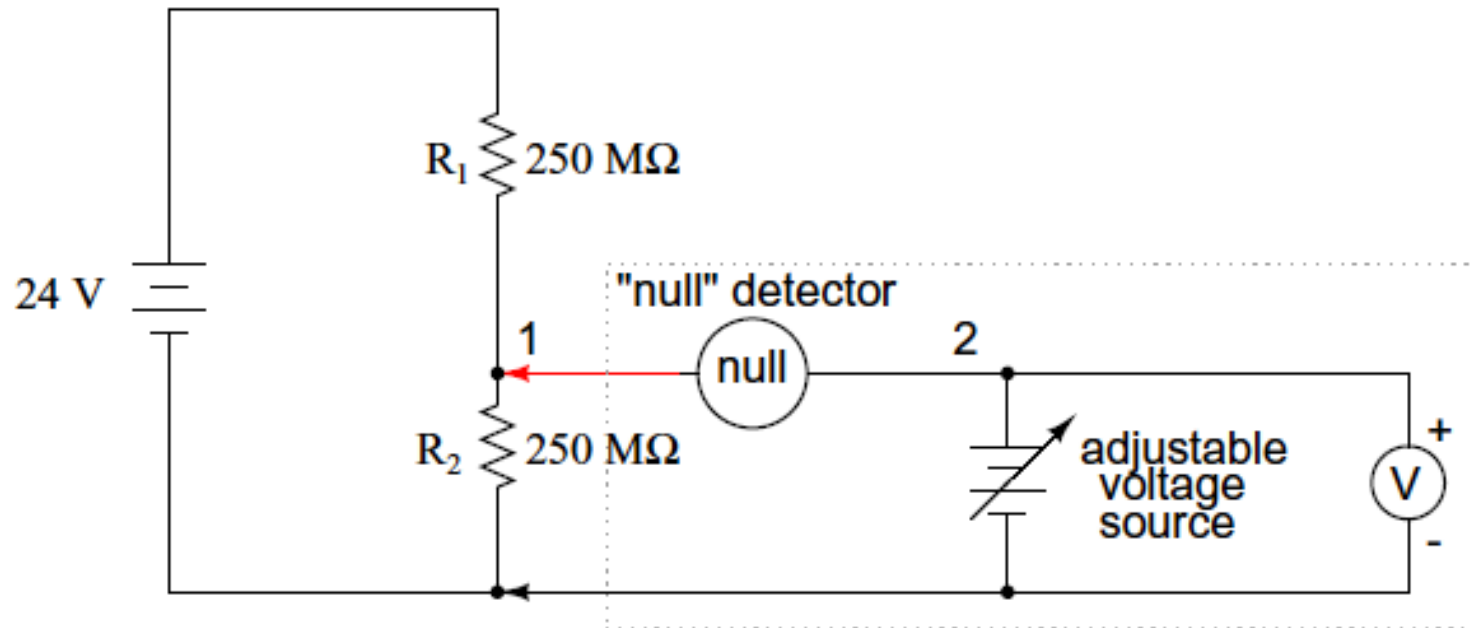
## Summary

- For accurate **current measurements**, the resistance of the current meter (connected in series with the circuit element under testing), must be **much smaller** than that of the tested element.
- For accurate **voltage measurements**, the resistance of the voltmeter (connected in parallel with the circuit element under testing), must be **much greater** than that of the tested element.
- The current and voltage measurement errors (relative) are given by:

$$\frac{\Delta I}{I_M} \approx \frac{I}{I_M} - 1 = \frac{R_A}{R}$$

$$\frac{\Delta V}{V_M} = \frac{V}{V_M} - 1 = \frac{R}{R_V}$$

## “Smart” voltmeter circuit – zero current consumption



*Adjust voltage source until null detector registers zero.  
Then, read voltmeter indication for voltage across  $R_2$ .*